

# Correction of zoomed morphology-based interpolation of contours

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**Abstract** - In medical imaging, a 3D object must often be reconstructed from serial cross-sections. Usually, the cross-sections are not closely spaced so that interpolation is needed to solve the problem of anisotropy. Different interpolation technique exist, applied either on slices either on objects.

Recently, a new contour interpolation method based on a zooming transform and on mathematical morphology has been developed. Thanks to the zooming transform, it works in any case. But, the obtained results are bigger and not so smooth as one could wish. This paper proposes a correction of the method in order to avoid so big results.

**Keywords** - interpolation, contour interpolation, image processing, 3D reconstruction, mathematical morphology.

## I. INTRODUCTION

In many medical applications, a 3D object must be reconstructed from serial cross-sections, either to aid in the comprehension of the object's structure or to facilitate its automatic manipulation and analysis. Major applications of this technique can be found in the study of the structure of biological specimens [1], the study of the structural and morphological characteristics of organs in biomedical sciences [2], radiation treatment planning and surgical planning [3].

Usually, the medical imagery system do not provide closely spaced cross-sections so that the spacing between the slices is much greater than the size of a pixel within the slice. Therefore, establishing some kind of interpolation between the slices is of vital importance for 3D reconstruction.

This interpolation step can be reached either by slice interpolation either by object interpolation. The slice interpolation was developed at the beginning of the 80's [4] but object interpolation is a more recent research axis (end of the 80's) which different groups are interested in [5-7].

Recently, a new contour interpolation method based on a zooming technique and on mathematical morphology has been developed [8]. Thanks to the zooming transform, it is a very interesting method because it works in any case, even with very complex shapes or with non overlapping contours or non crossing contours. But unfortunately, it gives bigger and not so smooth results as one could wish. This paper presents a correction of the method in the calculation of the inverse zooming factor in order to avoid so big results.

## II. METHOD

### A. Principle

Considering the source closed region  $R_S$  delimited by the source contour  $C_S$  and the target closed region  $R_T$  delimited by the target contour  $C_T$ , the principle of the proposed method is to apply a geodesic morphological dilation of the source region inside a zoomed version of the target region. Then, interpolated regions are computed by an appropriate reduction. This way of enlarging, interpolation processing, and reducing is called the zooming technique. The proposed method can be resumed in 5 steps :

- 1) operate a centroid registration transform which consists in superimposing  $R_S$  and  $R_T$ , according their centers of gravity,
- 2) zooming of  $R_T$  in such a way it includes  $R_S$ ,
- 3) process iteratively a geodesic dilation of  $R_S$  (until it corresponds to the zoomed version of  $R_T$ ),
- 4) inverse zooming of each result region of the iterative process,
- 5) operate the inverse centroid registration transform.

### B. Centroid Registration Transform

The aim of the centroid registration is to take into account the object translation for slice to slice in order to compensate for large shifts in object position. Firstly, the centroid  $c_S$  and  $c_T$  of each region  $R_S$  and  $R_T$  are computed. Then,  $R_S$  and  $R_T$  are transformed by translation in  $\tilde{R}_S$  and  $\tilde{R}_T$  respectively in order to superimpose the centroids  $c_S$  and  $c_T$  on a unique point  $O$  of the image [8] :

$$\tilde{R}_S = T\left(R_S, \vec{c_S O}\right) \quad \tilde{R}_T = T\left(R_T, \vec{c_T O}\right)$$

where  $T\left(R, \vec{U}\right)$  denotes the translation of a region  $R$  with a vector  $\vec{U}$ .

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### C. Zooming of the target region

The zooming transform consists in applying a scale coefficient or zooming factor to the region  $\tilde{R}_T$  in order to obtain the region  $\tilde{R}_T^*$  in such a way that it includes the region  $\tilde{R}_S$ . Let  $L_S$  (respectively  $L_T$ ) be the longer segment between  $c_S$  (respectively  $c_T$ ) and a point belonging to  $C_S$  (respectively  $C_T$ ) [8]. Let  $S_S$  (respectively  $S_T$ ) be the shorter segment between  $c_S$  (respectively  $c_T$ ) and a point belonging to  $C_S$  (respectively  $C_T$ ) [8]. The zooming factor  $K$  is computed as follows :

$$K = \frac{L_S}{S_T}$$

This process defines the transformation  $Z$  which leads to the computation of :

$$\tilde{R}_T^* = Z(\tilde{R}_T, K)$$

where  $Z(R, \tau)$  denotes the enlargement provided by the zooming transform with a factor  $\tau$ .

### D. Interpolation process

The main stage of the presented interpolation method is the iterative process which deforms progressively  $\tilde{R}_S$  into  $\tilde{R}_T^*$ . It is raised by applying a geodesic morphological dilation of  $\tilde{R}_S$  inside  $\tilde{R}_T^*$ .

Let  $B_X(x, r)$  be a geodesic neighbourhood ball centred at  $x$  and having  $r$  for radius, considered in the region  $X$  of the plane ; it is defined as :

$$B_X(x, r) = \{y \in X, d_X(x, y) \leq r\}$$

where  $d_X(x, y)$  denotes the geodesic distance between  $x$  and  $y$  in the region  $X$ . Then, the geodesic dilation of  $Y$  in  $X$  with a radius  $r$  is defined as :

$$D_X^R(Y) = \{x, B_X(x, r) \cap Y \neq \emptyset\} = \{x, d_X(x, Y) \leq r\}$$

In our case,  $X = \tilde{R}_T^*$ ,  $Y = \tilde{R}_S$  and  $r=1$ .

Let  $N_R$  be the number of iterations taken by the iterative process to reach  $\tilde{R}_T^*$ ,  $N_R$  regions  $\tilde{R}_i^*$   $i=1, \dots, N_R$  are obtained. For a given  $\alpha$  ( $0 \leq \alpha \leq 1$ ), the corresponding region is  $\tilde{R}_\alpha^*$  and depends on  $\alpha$  ( $\tilde{R}_0^* = \tilde{R}_S$  and  $\tilde{R}_1^* = \tilde{R}_T^*$ ). It is given by :  $\tilde{R}_\alpha^* = \tilde{R}_I^*(\alpha)$  with  $I(\alpha) = E(\alpha N_R)$  where  $E(\cdot)$  denotes the integer part function.

This process, described above, defines the interpolation  $\tilde{I}_R$  for the computation of  $\tilde{R}_\alpha^*$  from  $\tilde{R}_S, \tilde{R}_T^*, \alpha$  :

$$\tilde{R}_\alpha^* = \tilde{I}_R(\tilde{R}_S, \tilde{R}_T^*, \alpha)$$

### E. Inverse zooming of the selected dilated regions

After the interpolation step, each region  $\tilde{R}_\alpha^*$  must be resized correctly in function of  $\alpha$ . It is the object of the inverse zooming transform which consists in applying a scale coefficient or inverse zooming factor  $J$  given by :

$$J(\alpha) = \frac{1-K}{K} \alpha + 1 \quad (*)$$

The region  $\tilde{R}_\alpha$  is then obtained from  $\tilde{R}_\alpha^*$  by :

$$\tilde{R}_\alpha = Z(\tilde{R}_\alpha^*, J)$$

### F. Inverse Centroid Registration Transform

After the interpolation process, an interpolated region is obtained for each  $\alpha$ . Then, each real interpolated region  $R_\alpha$  is computed by taking into account the inverse centroid registration transform which consists in computing the right position of the interpolated region :

$$R_\alpha = T\left(\tilde{R}_\alpha, (1-\alpha)\vec{Oc}_S + \alpha\vec{Oc}_T\right)$$

## III. LIMITATION

Thanks to the zooming transform, the described method is interesting because it works in any case even with complex shapes. Moreover, it does not suffer from fixed points phenomenon or non crossing contours. But unfortunately, the obtained results are bigger and not so smooth as one could wish as illustrated in fig. 1 and fig. 2. Fig. 1 represents two original contours to be interpolated, superimposed on a unique image. Fig. 2 represents the superimposed results after the computation of three interpolated contours.

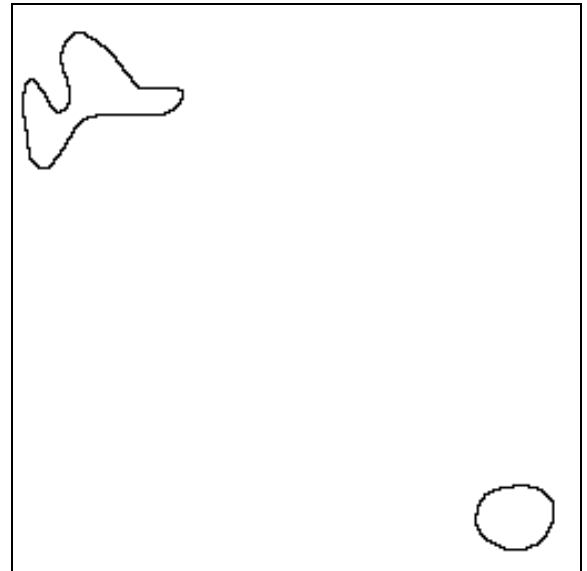


Fig. 1 : two original contours superimposed on the same image

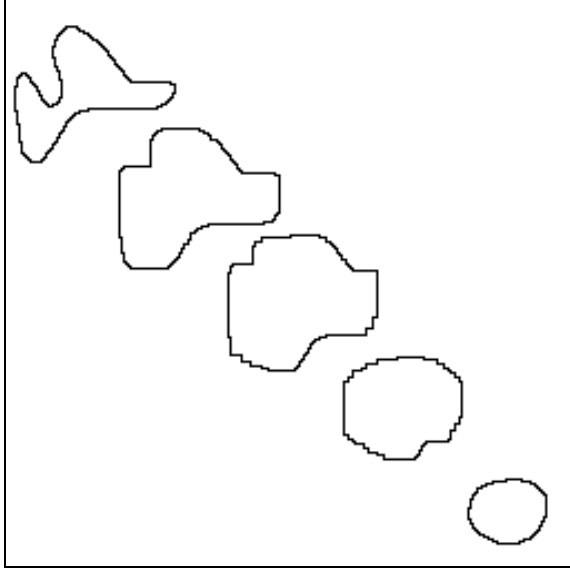


Fig. 2 : Superimposition of the results obtained with the original method

The main reason is the unique speed of deformation for each pair of implicitly matched points along the geodesic paths reaching them. Thus, the target contour is not reached at each point at the same time instead of being reached at each point at the last time of the iterative process [8].

An other reason which explains the size problem is that the calculation of the inverse zooming factor is not satisfying and the object of this paper is to propose an other formula to compute it.

#### IV. CORRECTION AND RESULTS

The inverse zooming factor must verify  $J(0)=1$  and  $J(1)=1/K$ . In the previous version, the inverse zooming factor was searched as  $J(\alpha)=A\alpha + B$  and led to formula (\*).

But, it is incorrect. The best way to proof it is to take a simple example. Suppose two identical circles as  $C_S$  and  $C_T$ . Suppose their radius equal to 1 and that the zooming factor  $K$  is chosen to be 2. If  $\alpha = 1/2$ , the corresponding contour after the iterative process is a circle with a radius equal to  $3/2$  (fig. 3). With the previous formula (\*), the inverse zooming factor  $J$  for  $\alpha = 1/2$  is  $3/4$  instead of  $2/3$ .

To propose an other calculation of  $J$ , let us consider the region  $\tilde{R}_\alpha^*$  for a given  $\alpha$ . This region can be expressed as follows :

$$\tilde{R}_\alpha^* = (1-\alpha)\tilde{R}_S + \alpha\tilde{R}_T^*$$

Thus, the zooming factor  $K(\alpha)$  corresponding to  $\tilde{R}_\alpha^*$  is given by :

$$K(\alpha) = (1-\alpha) + \alpha K$$

and the inverse zooming factor defined by  $J(\alpha) = 1/K(\alpha)$  is

$$\text{then : } J(\alpha) = \frac{1}{(1-\alpha) + \alpha K} \quad (**)$$

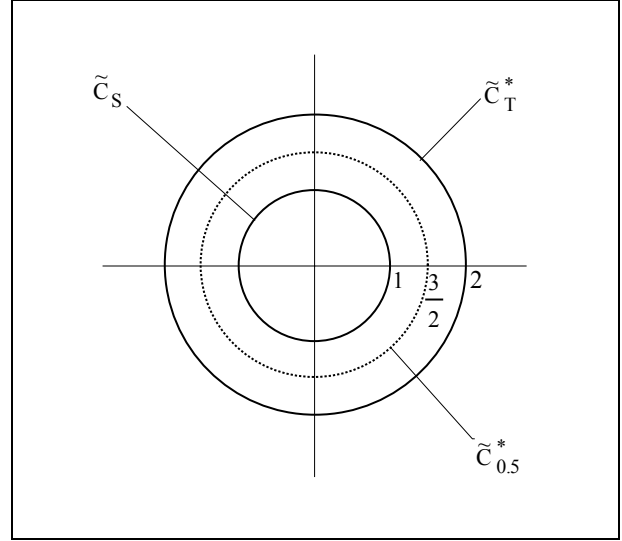


Fig. 3 : Example of two identical circles and an interpolated contour at  $\alpha=1/2$

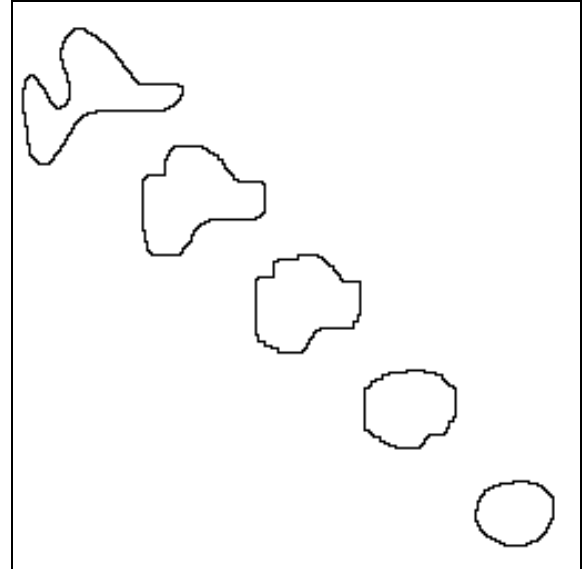


Fig. 4 : Superimposition of the results obtained with the corrected method

It can be verified that  $J(0)=1$ ,  $J(1)=1/K$  and that for the previous example of the two identical circles  $J(1/2)=2/3$ .

Fig. 4 shows the results obtained by taking into account this new formula (\*\*) for the calculation of inverse zooming factor. The contours are not so big than with the previous method.

#### V. CONCLUSION AND PERSPECTIVES

Recently, a method of contour interpolation based on a zooming transform and on geodesic morphological dilation has been developed. It gives interesting results because it works in any complex case but the results are bigger and not so smooth as one could wish. A correction is presented in this paper in order to limit the drawback of big size phenomenon.

The interest of this method is the use of the zooming transform which allows applying it on any complex shape. The perspective will consist in solving the problem of smoothness by developing an interpolation process with adapted speed of deformation.

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